RIZVI COLLEGE OF ARTS, SCIENCE & COMMERCE
OFF CARTER ROAD, BANDRA (WEST)

DETARTMENT OF PHYSICS

**F.Y.B.SC. [ PHYSICS ]**
**SEM II**
**PAPER I - UNIT III**

# WAVE MOTION

## Part 2

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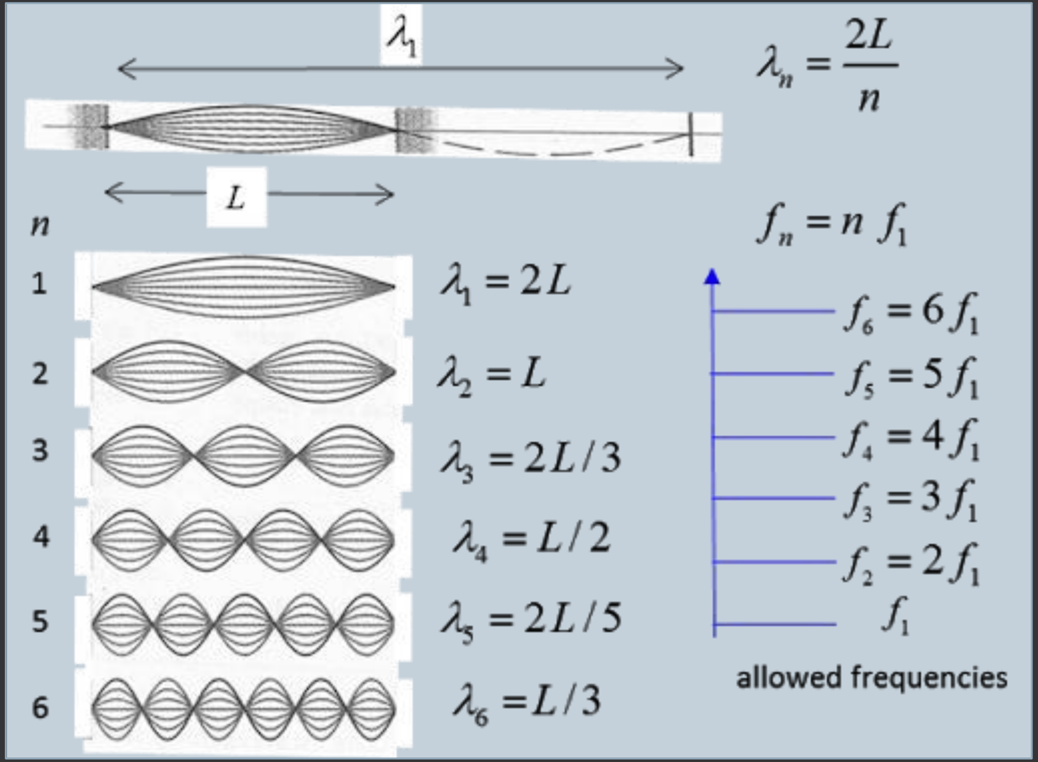
## Contents of part 2

- Normal modes of a string
- Group velocity
- Phase velocity
- Plane waves

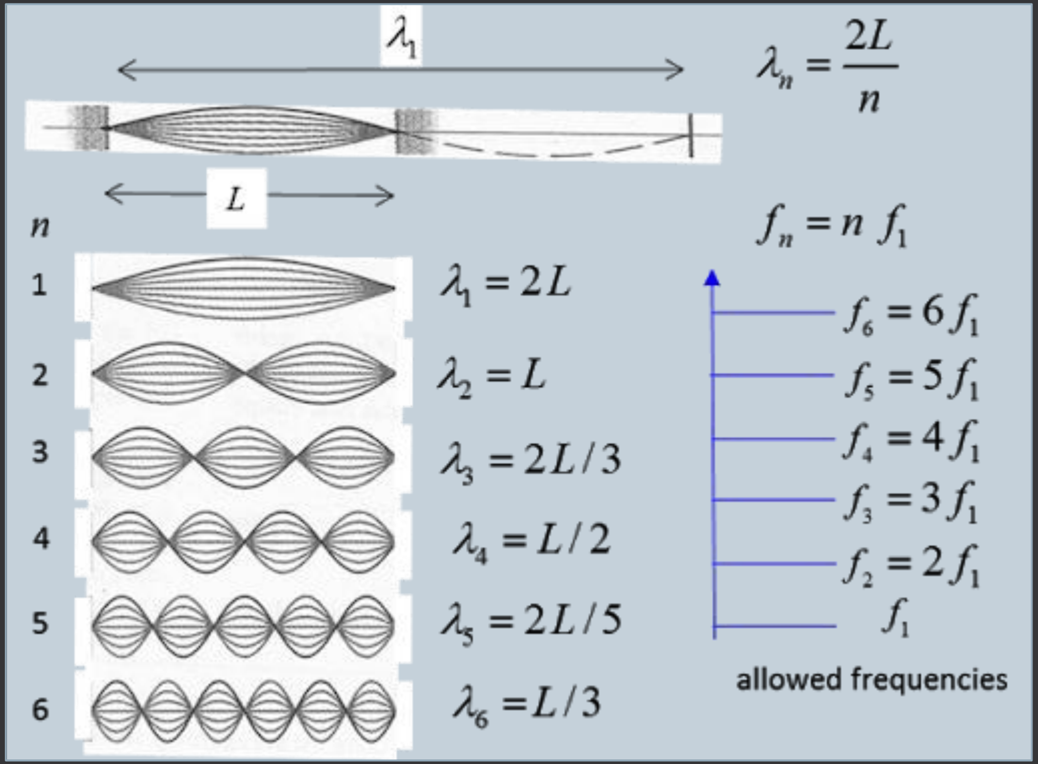
# NORMAL MODES OF A STRING

- Consider a string of definite length L fixed at both the ends with rigid support.
- The production of sound wave on a such stretched string will reflect the sound from both the ends.
- **"Nodes"** are produced at both the ends and **"Antinodes"** are at the center.
- The distance between the adjacent nodes is $\frac{\lambda}{2}$ …..( See the diagram)



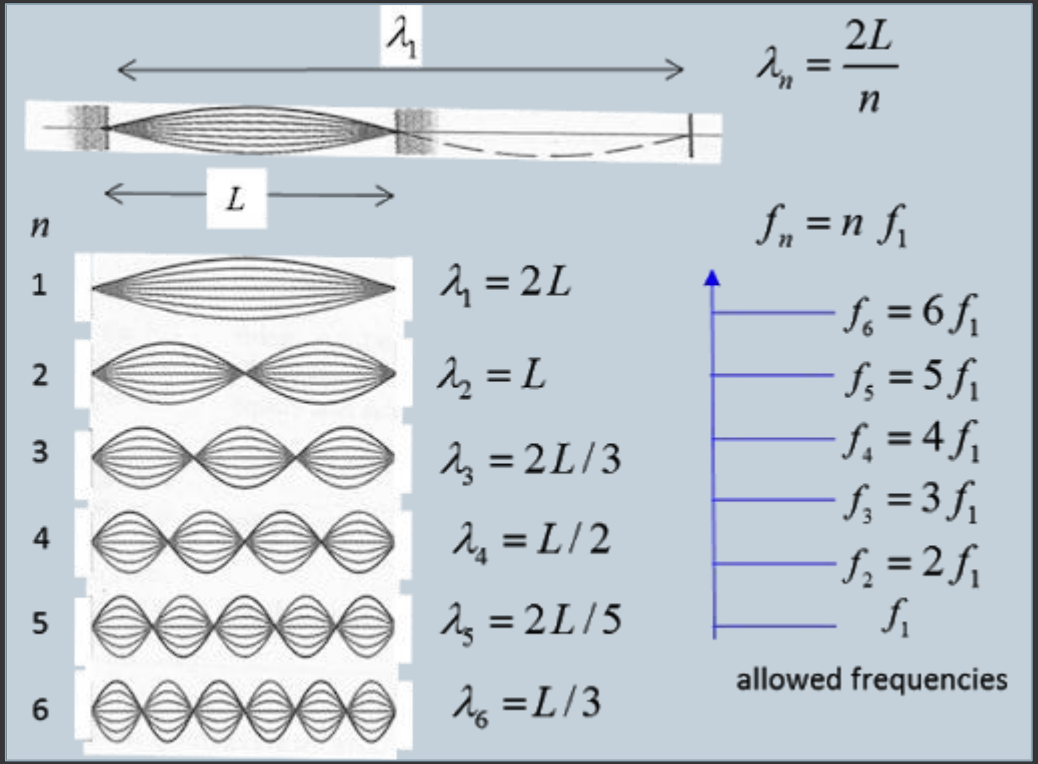
# NORMAL MODES OF A STRING

- Length of the string is then $\frac{\lambda}{2}, 2\frac{\lambda}{2}, 3\frac{\lambda}{2}$...... Or in general it is some integer number of wavelength; $L = n\frac{\lambda}{2}$ ; where n = 1,2,3,4....
- The standing wave can exist in a string with length L only when its wavelength is; $\lambda_n = \frac{2L}{n}$ where n = 1,2,3,4....



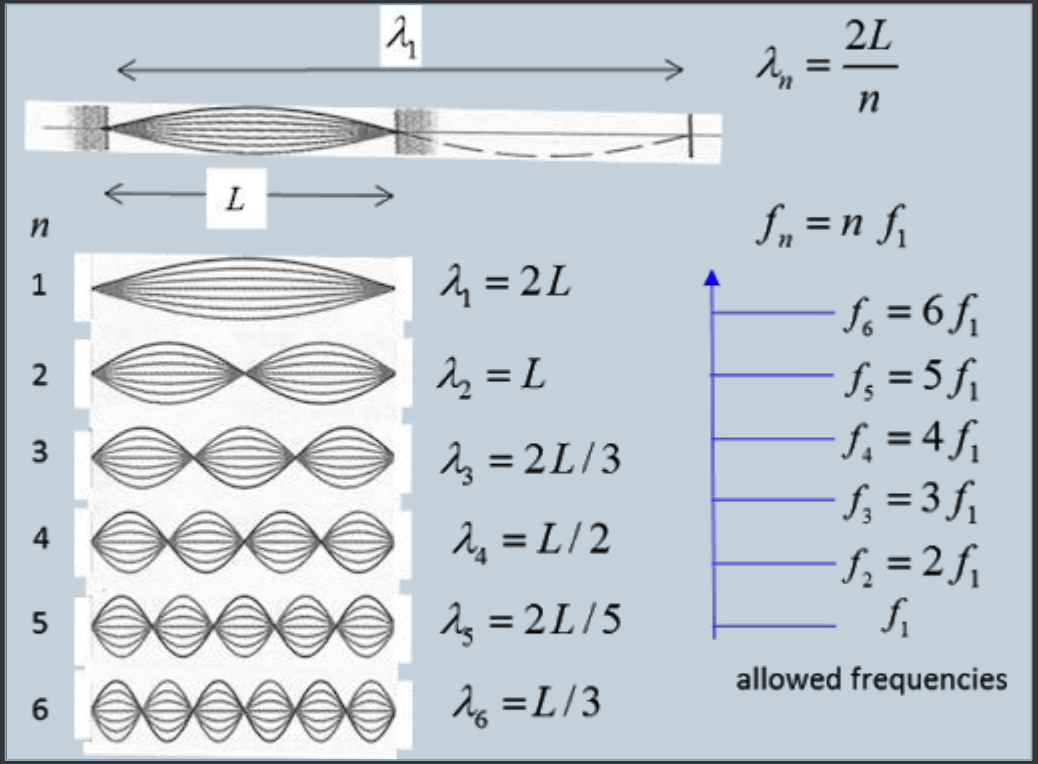
# NORMAL MODES OF A STRING

- Standing waves are possible only when wavelengths are equal to one of these values.
- The possible frequencies corresponding to each of these wavelengths are; $f_1 = \frac{c}{2L}$ , $f_2 = \frac{2c}{2L}$ ….. In general the frequency formula will be ; $f_n = \frac{nc}{2L}$ ; where n = 1,2,3,4….
- $f_1 = \frac{c}{2L}$ is called as Fundamental frequency. And all other frequencies are integer multiple of $f_1$ such as 2 $f_1$ ,3 $f_1$ , 4 $f_1$ …..n $f_1$



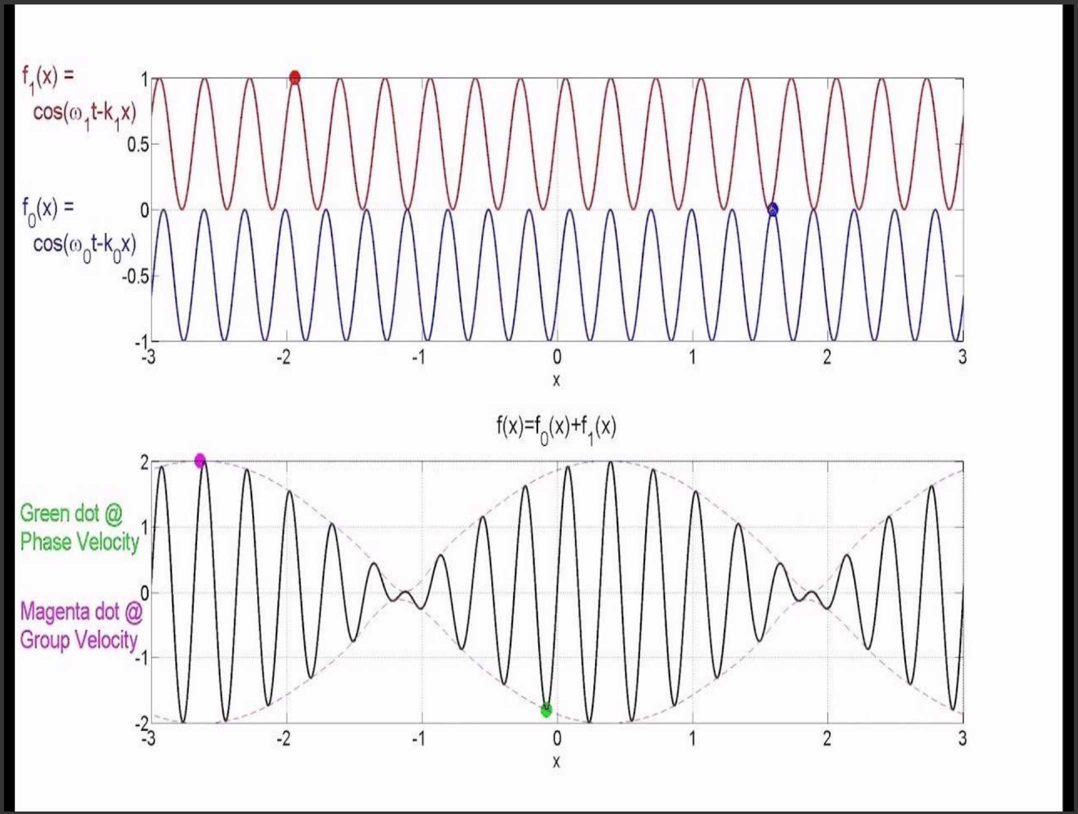
# NORMAL MODES OF A STRING

- These frequencies are called as **Harmonics** **and series is called as Harmonic series.**
- In music $f_2$, $f_3$ are called overtones; $f_2$ is second harmonic or first overtone, $f_3$ is third harmonic or second overtone.
- In music superposition of normal modes are present simultaneously to produce harmony.



# GROUP VELOCITY

1. A group of to individual waves with slight difference in their wavelength is shown in the diagram.
2. The superposition of these waves is shown in the diagram below it.
3. The amplitude of resultant consist of number of components waves with the difference in wavelengths. The resultant is known as Group of waves or a wave packet.
4. The packet travels with a velocity in a medium which is different from the velocity of the individual wave. Therefore, the velocity of wave packet is called as group velocity.



# GROUP VELOCITY

Derivation for determination of group Velocity.

Consider two travelling waves with slightly different frequencies $\omega_1$ and $\omega_2$ and wave numbers $k_1$ & $k_2$.

$$y_1 = A\sin(\omega_1 t - k_1 x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

$$y_2 = A\sin(\omega_2 t - k_2 x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Using Superposition principle; $y = y_1 + y_2$

$$y = A\sin(\omega_1 t - k_1 x) + A\sin(\omega_2 t - k_2 x)$$

$$y = A[\sin(\omega_1 t - k_1 x) + \sin(\omega_2 t - k_2 x)] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$ **... trigonometric rule**

$$y = 2A\sin\left[\frac{(\omega_1+\omega_2)}{2}t - \frac{(k_1+k_2)}{2}x\right]\cos\left[\frac{(\omega_1-\omega_2)}{2}t - \frac{(k_1-k_2)}{2}x\right]$$

Let $\frac{(\omega_1+\omega_2)}{2} = \omega$ $\quad \frac{(k_1+k_2)}{2} = k$ $\quad \frac{(\omega_1-\omega_2)}{2} = \Delta\omega$ $\quad \frac{(k_1-k_2)}{2} = \Delta k$

# GROUP VELOCITY

Derivation for determination of group Velocity.

$$y = 2A\sin[\omega t - kx]\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]$$

$$y = 2A\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]\sin[\omega t - kx]$$

Amplitude of the wave is $2A\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]$

- ✓ Thus the velocity with which the wave3 packet obtained due to superposition of wave travelling in a group is called group velocity.
- ✓ A wave packet is a group of several waves of slightly different velocity and different wavelength.
- ✓ For the amplitude of the wave ; $\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x = 0$ ; $V_g = \frac{\frac{\Delta\omega}{2}}{\frac{\Delta k}{2}} = \frac{\Delta\omega}{\Delta k}$

Thus the group velocity, when $\frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$ is $V_g = \frac{d\omega}{dk}$

The expression is valid for the whole spectrum of superposing waves.

# Phase Velocity

1. It is the rate at which the phase of the wave propagates in space. This is the velocity at which the phase of any one frequency component of the wave travels.
2. The phase velocity is given in terms of wavelength and the period; $V_p = \frac{\lambda}{T}$
3. Equivalently in terms of wave's angular frequency ω and wave vector k ; $V_p = \frac{\omega}{k}$ ; $\omega = \frac{2\pi}{T}$ $and\ k = \frac{2\pi}{\lambda}$
4. Thus the component waves of a wave packet is called phase velocity . the phase $\omega t - kx = constant$

$$\therefore \ \boldsymbol{\omega\, dt = k dx \qquad \frac{dx}{dt} = \frac{\omega}{k} = V_p}$$

# Phase Velocity

DISPERSIVE MEDIUM:

A medium in which wave packet loses its initial shape is called dispersive medium. Most media in nature are dispersive.

$$\omega = kV_p \quad V_g = \frac{d\omega}{dk} = \frac{d\,(kV_p)}{dk} = V_p + k\frac{d\,(V_p)}{dk}$$

$$\therefore V_g = V_p + \left(\frac{2\pi}{\lambda}\right)\frac{d\,(V_p)}{d\left(\frac{2\pi}{\lambda}\right)}$$

$$\therefore V_g = V_p + \left(\frac{1}{\lambda}\right)\frac{d\,(V_p)}{d\left(\frac{1}{\lambda}\right)}$$

Using chain rule $d\left(\frac{1}{\lambda}\right) = -\frac{1}{\lambda^2}\,d\lambda$

$$\therefore \boldsymbol{V_g = V_p - \frac{d\,V_p}{d}}$$

# PLANE WAVES

1. Any small portion of a spherical wavefront that is far from the source is considered to be as plane wavefront.
2. A plane wave is awave for which the phase has the same value at all the points on an infinite plane.
3. It is a uniform plane wave as the amplitude remain constant.
4. Plane wave vary only in the direction of propagation and are uniform in planes to the direction of propagation.
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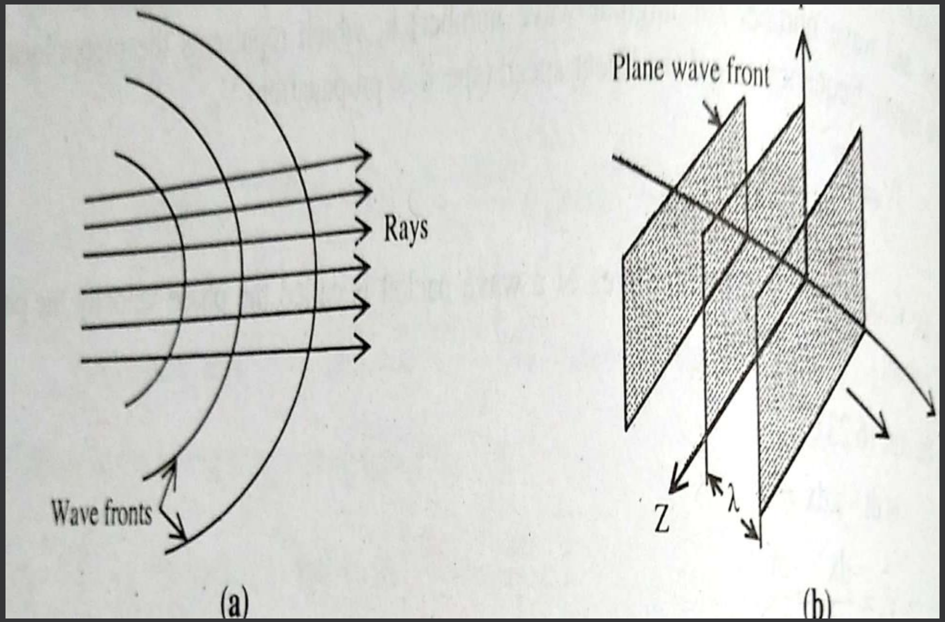


# PLANE WAVES

Expression for the Plane Waves:

$$y = Asin\,(kx \pm \omega t) = Ae^{i(kx \pm \omega t)}$$

Using Euler's formula ; $e^{i\theta} = \cos\theta + i\,sin\theta$

$$y = Ae^{i(kx - \omega t)} \quad \text{and} \quad y = Ae^{-i(\omega t - kx)}$$

Thus complex form of the wave is split up into space and time coordinates.

$$y = A\left(e^{-i\omega t}\,e^{ikx}\right)$$

$Ae^{ikx}$ *represents complex form of the amplitude of the wave*

$Ae^{-i\omega t}$ *represents harmonic disturbance.*

1. Certain radar emits 9400-MHz radio waves in groups 0.08 μs in duration. The time needed for these groups to reach a target, be reflected and return back to the radar is indicative of the distance of the target. The velocity of these waves, like other electromagnetic waves Find (c) the wavelength of these waves, (d) the length of each wave group, which governs how precisely the radar can measure distances.

(c) Since, 1 MHz = $10^6$ Hz,

$$9400\ MHz = 9.4 \times 10^9\ Hz$$

Therefore, the wavelength

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8\ m/s}{9.4 \times 10^9\ Hz} = 3.19 \times 10^{-2}\ m$$

(d) The length $s$ of each wave group is

$$s = ct = (3 \times 10^8\ m/s)(8 \times 10^{-8}\ s) = 24\ m$$

(e) There are two ways to find the number of waves $n$ in each group:

$$n = ft = (9.4 \times 10^9\ Hz)(8 \times 10^{-8}\ s) = 752\ waves$$

2. A wave equation is represented by $y = 8\sin(10x - 10t)$ Determine the amplitude Wavelength angular frequency wave number and velocity of the wave.

**Solution:**

Comparing the given wave equation with equation (10.9), we find that the wave is travelling in the positive x-direction, with amplitude A = 8 cm, angular frequency $\omega = 10\, rad/s$ and the wave number k = 10 rad/cm.

From the definition of the wave number, we have

$$k = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{10} = 0.63\, cm$$

Further, using equation (10.10), we have

$$v = \frac{\omega}{k} = \frac{10\, rad/s}{10\, rad/cm} = 1\, cm/s$$

3. A transverse wave is travelling along the string from left to right. The figure below represents the shape of the string at a given instant. At this instant ( a) which points have upward velocity? ( b) Which points have downward velocity? ( c) Which points have zero velocity? (d) Which points have maximum of velocity?

For a wave travelling in positive x-direction, the particle velocity $v_p$ at any instant is given by

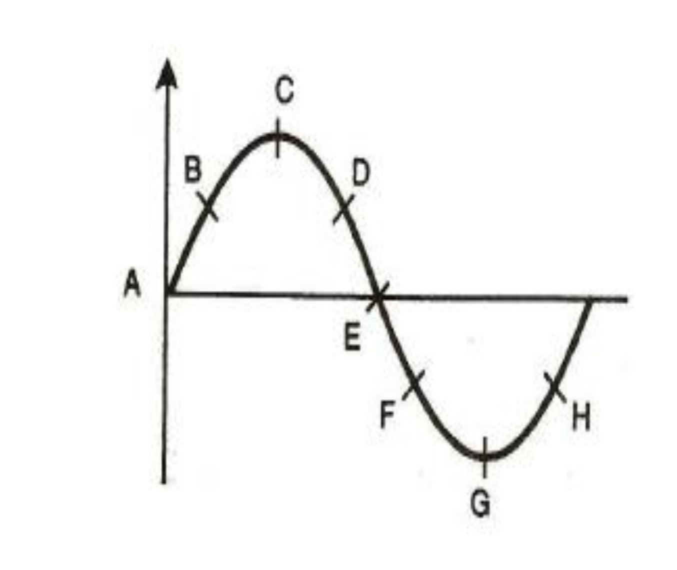
$$v_p = \left(\frac{dy}{dt}\right)_x$$

$$\Rightarrow v_p = -\omega A \cos(kx - \omega t)$$

Further, the slope of the wave is given as

$$\frac{dy}{dx} = Ak\cos(kx - \omega t)$$

$$v_p = -\frac{\omega}{k} \times (slope) = -v \times (slope)$$



(a) For upward velocity, $v_p$ = *positive*, so the slope must be negative which is at points D, E and F.

(b) For downward velocity, $v_p$ = *negative*, so the slope must be positive which is at points A, B and H.

(c) For zero velocity, the slope must be zero which is at C and G.

(d) For maximum magnitude of velocity, $|slope| = maximum$ which is at A and E.

Thank you