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DETARTMENT OF PHYSICS

F.Y.B.SC. [PHYSICS]
SEM II
PAPER II - UNIT II

Circuit theorems

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CONTENT

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3. Examples of Thevenin theorem
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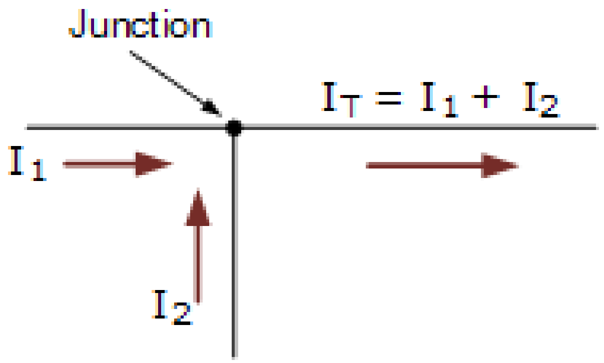
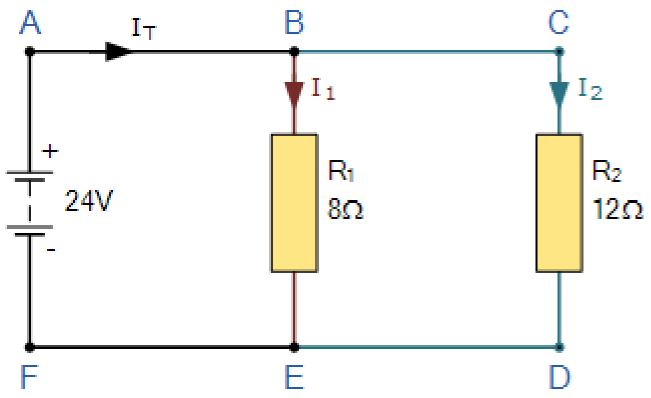
Kirchhoff’s Current Law

- To determine the amount or magnitude of the electrical current flowing around an electrical or electronic circuit, we need to use certain laws or rules that allows us to write down these currents in the form of an equation. The network equations used are those according to Kirchhoff’s laws, and as we are dealing with circuit currents, we will be looking at Kirchhoff’s current law, (KCL).
- **Gustav Kirchhoff’s Current Law** is one of the fundamental laws used for circuit analysis. His current law states that for a parallel path **the total current entering a circuits junction is exactly equal to the total current leaving the same junction**. This is because it has no other place to go as no charge is lost.
- In other words the algebraic sum of ALL the currents entering and leaving a junction must be equal to zero as: $\Sigma I_{IN} = \Sigma I_{OUT}$.
- This idea by Kirchhoff is commonly known as the **Conservation of Charge**, as the current is conserved around the junction with no loss of current. Lets look at a simple example of Kirchhoff’s current law (KCL) when applied to a single junction.

- Here in this simple single junction example, the current I_T leaving the junction is the algebraic sum of the two currents, I_1 and I_2 entering the same junction. That is $I_T = I_1 + I_2$.
- Note that we could also write this correctly as the algebraic sum of: $I_T - (I_1 + I_2) = 0$.
- So if I_1 equals 3 amperes and I_2 is equal to 2 amperes, then the total current, I_T leaving the junction will be 3 + 2 = 5 amperes, and we can use this basic law for any number of junctions or nodes as the sum of the currents both entering and leaving will be the same.
- Also, if we reversed the directions of the currents, the resulting equations would still hold true for I_1 or I_2. As $I_1 = I_T - I_2 = 5 - 2 = 3$ amps, and $I_2 = I_T - I_1 = 5 - 3 = 2$ amps. Thus we can think of the currents entering the junction as being positive (+), while the ones leaving the junction as being negative (-).
- Then we can see that the mathematical sum of the currents either entering or leaving the junction and in whatever direction will always be equal to zero, and this forms the basis of Kirchhoff's Junction Rule, more commonly known as *Kirchhoff's Current Law*, or (KCL).



Resistors in Parallel



To start, all the current, I_T leaves the 24 volt supply and arrives at point A and from there it enters node B. Node B is a junction as the current can now split into two distinct directions, with some of the current flowing downwards and through resistor R_1 with the remainder continuing on through resistor R_2 via node C. Note that the currents flowing into and out of a node point are commonly called branch currents.

We can use Ohm's Law to determine the individual branch currents through each resistor as: I = V/R, thus:

For current branch B to E through resistor R_1

$$I_{B-E} = I_1 = \frac{V}{R_1} = \frac{24}{8} = 3A$$

For current branch C to D through resistor R_2

$$I_{C-D} = I_2 = \frac{V}{R_2} = \frac{24}{12} = 2A$$

- From above we know that Kirchhoff’s current law states that the sum of the currents entering a junction must equal the sum of the currents leaving the junction, and in our simple example above, there is one current, I_T going into the junction at node B and two currents leaving the junction, I_1 and I_2.
- Since we now know from calculation that the currents leaving the junction at node B is I_1 equals 3 amps and I_2 equals 2 amps, the sum of the currents entering the junction at node B must equal 3 + 2 = 5 amps. Thus $\Sigma_{IN} = I_T = 5$ amperes.
- In our example, we have two distinct junctions at node B and node E, thus we can confirm this value for I_T as the two currents recombine again at node E. So, for Kirchhoff’s junction rule to hold true, the sum of the currents into point F must equal the sum of the currents flowing out of the junction at node E.
- As the two currents entering junction E are 3 amps and 2 amps respectively, the sum of the currents entering point F is therefore: 3 + 2 = 5 amperes. Thus $\Sigma_{IN} = I_T =$ 5 amperes and therefore Kirchhoff’s current law holds true as this is the same value as the current leaving point A.

Kirchhoff's Voltage law

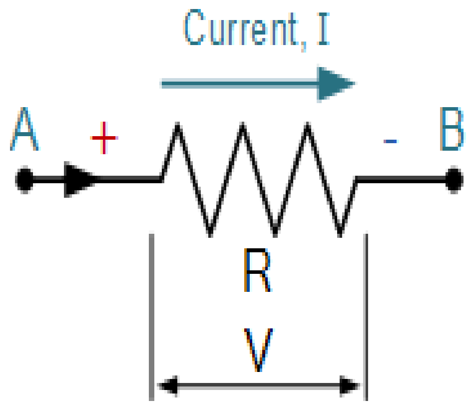
Gustav Kirchhoff's Voltage Law is the second of his fundamental laws we can use for circuit analysis. His voltage law states that for a closed loop series path **the algebraic sum of all the voltages around any closed loop in a circuit is equal to zero**. This is because a circuit loop is a closed conducting path so no energy is lost.

In other words the algebraic sum of ALL the potential differences around the loop must be equal to zero as: **$\Sigma V = 0$.** Note here that the term "algebraic sum" means to take into account the polarities and signs of the sources and voltage drops around the loop.

This idea by Kirchhoff is commonly known as the Conservation of Energy, as moving around a closed loop, or circuit, you will end up back to where you started in the circuit and therefore back to the same initial potential with no loss of voltage around the loop. Hence any voltage drops around the loop must be equal to any voltage sources met along the way.

Understanding KVL

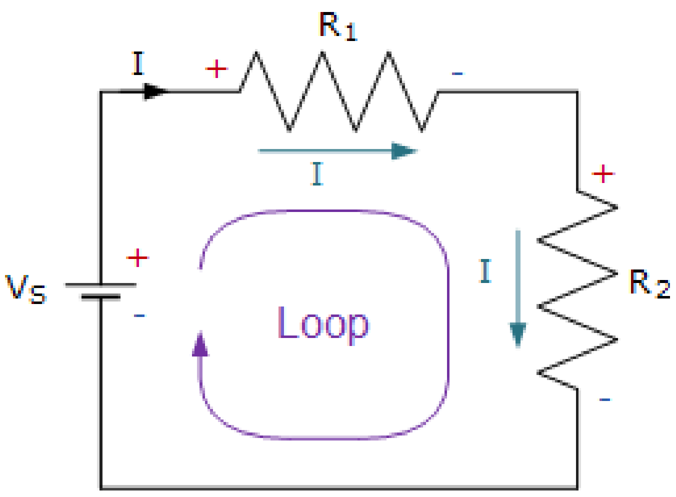
- Assume that the current, I is in the same direction as the flow of positive charge, that is conventional current flow.
- The flow of current through the resistor is from point A to point B, that is from positive terminal to a negative terminal.
- **Thus as we are travelling in the same direction as current flow, there will be a *fall* in potential across the resistive element giving rise to a -IR voltage drop across it.**



- If the flow of current was in the opposite direction from point B to point A, then there would be a *rise* in potential across the resistive element as we are moving from a - potential to a + potential giving us a +I*R voltage drop.

As a general rule: You will loose potential in the same direction of current across an element and gain potential as you move in the direction of an emf source.

A Single Circuit Loop



- Kirchhoff's voltage law states that the algebraic sum of the potential differences in any loop must be equal to zero as: ΣV = 0. Since the two resistors, R_1 and R_2 are wired together in a series connection, they are both part of the same loop so the same current must flow through each resistor.
- Thus the voltage drop across resistor, $R_1 = I*R_1$ and the voltage drop across resistor, $R_2 = I*R_2$ giving by KVL:

$$V_S + (-IR_1) + (-IR_2) = 0$$

$$\therefore V_S = IR_1 + IR_2$$

$$V_S = I(R_1 + R_2)$$

$$V_S = IR_T$$

$$\text{Where: } R_T = R_1 + R_2$$

We can see that applying Kirchhoff's Voltage Law to this single closed loop produces the formula for the equivalent or total resistance in the series circuit and we can expand on this to find the values of the voltage drops around the loop.

$$R_T = R_1 + R_2$$

$$I = \frac{V_S}{R_T} = \frac{V_S}{R_1 + R_2}$$

$$V_{R1} = IR_1 = V_S\left(\frac{R_1}{R_1 + R_2}\right)$$

$$V_{R2} = IR_2 = V_S\left(\frac{R_2}{R_1 + R_2}\right)$$

Kirchhoff's Voltage Law Example No1

Three resistor 10 ohms, 20 ohms and 30 ohms, respectively are connected in series across a 12 volt battery supply. Calculate: a) the total resistance, b) the circuit current, c) the current through each resistor, d) the voltage drop across each resistor, e) verify that Kirchhoff's voltage law, KVL holds true.

a) Total Resistance (R_T)

$$R_T = R_1 + R_2 + R_3 = 10\Omega + 20\Omega + 30\Omega = 60\Omega$$

Then the total circuit resistance R_T is equal to 60Ω

b) Circuit Current (I)

$$I = \frac{V_S}{R_T} = \frac{12}{60} = 0.2A$$

c) Current Through Each Resistor

The resistors are wired together in series, they are all part of the same loop and therefore each experience the same amount of current. Thus:

$$I_{R1} = I_{R2} = I_{R3} = I_{SERIES} = 0.2 \text{ amperes}$$

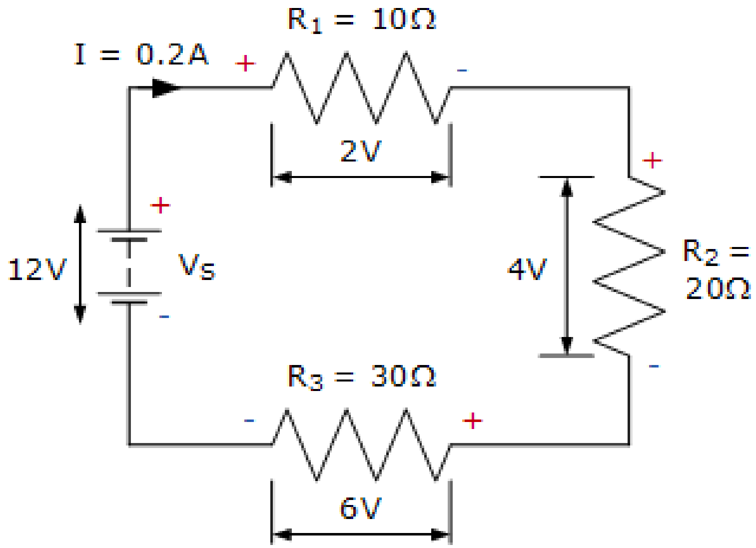
Verifying Kirchhoff's Voltage Law

$$V_S + (-IR_1) + (-IR_2) + (-IR_3) = 0$$

$$12 + (-0.2\times10) + (-0.2\times20) + (-0.2\times30) = 0$$

$$12 + (-2) + (-4) + (-6) = 0$$

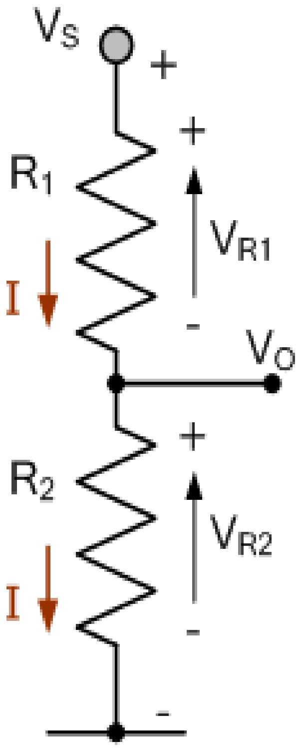
$$\therefore 12 - 2 - 4 - 6 = 0$$



Thus Kirchhoff's voltage law holds true as the individual voltage drops around the closed loop add up to the total.

Resistive Voltage Divider Circuit

- Here the circuit consists of two resistors connected together in series: R_1, and R_2. Since the two resistors are connected in series, it must therefore follow that the same value of electric current must flow through each resistive element of the circuit as it has nowhere else to go. Thus providing an I*R voltage drop across each resistive element.
- With a supply or source voltage, V_S applied across this series combination, we can apply Kirchhoff's Voltage Law, (KVL) and also using Ohm's Law to find the voltage dropped across each resistor derived in terms of the common current, I flowing through them. So solving for the current (I) flowing through the series network gives us:



$$V_S = V_{R1} + V_{R2} \quad \text{(KVL)}$$

$$V_{R1} = I \times R_1 \quad \text{and} \quad V_{R2} = I \times R_2$$

$$\text{Then: } V_S = I \times R_1 + I \times R_2$$

$$\therefore V_S = I(R_1 + R_2)$$

$$\text{So: } I = \frac{V_S}{(R_1 + R_2)}$$

The current flowing through the series network is simply I = V/R following Ohm's Law. Since the current is common to both resistors, ($I_{R1} = I_{R2}$) we can calculate the voltage dropped across resistor, R_2 in the above series circuit as being:

$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{V_S}{\left(R_1 + R_2\right)}$$

$$\therefore\ V_{R2} = V_S\left(\frac{R_2}{R_1 + R_2}\right)$$

Likewise for resistor R_1 as being:

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{V_S}{\left(R_1 + R_2\right)}$$

$$\therefore\ V_{R1} = V_S\left(\frac{R_1}{R_1 + R_2}\right)$$

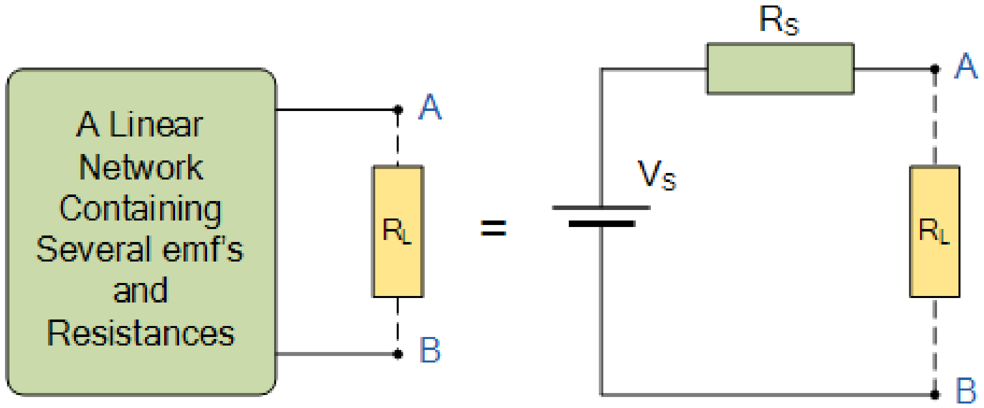
Voltage Divider Equation

$$V_{R(x)} = V_S \left(\frac{R_X}{R_T} \right)$$

Where: $V_{R(x)}$ is the voltage drop across the resistor, R_X is the value of the resistor, and R_T is the total resistance of the series network. This voltage divider equation can be used for any number of series resistances connected together because of the proportional relationship between each resistance, R and its corresponding voltage drop, V. Note however, that this equation is given for an unloaded *voltage divider network* without any additional resistive load connected or parallel branch currents.

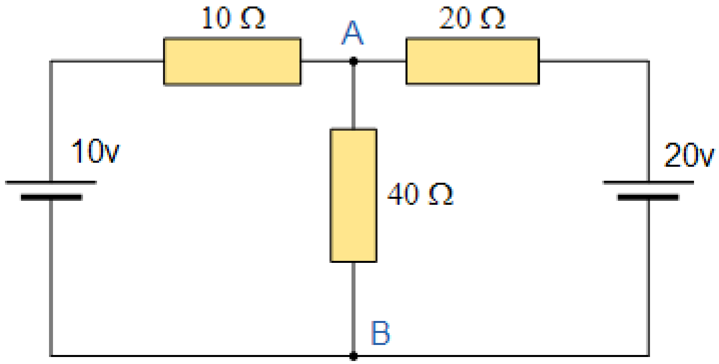
Thevenin’s Theorem.

Thevenin’s Theorem states that ***“Any linear circuit containing several voltages and resistances can be replaced by just one single voltage in series with a single resistance connected across the load“.***
In other words, it is possible to simplify any electrical circuit, no matter how complex, to an equivalent two-terminal circuit with just a single constant voltage source in series with a resistance (or impedance) connected to a load as shown below.

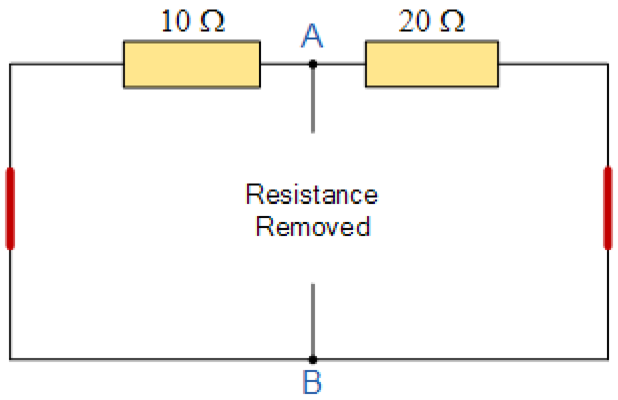


As far as the load resistor R_L is concerned, any complex “one-port” network consisting of multiple resistive circuit elements and energy sources can be replaced by one single equivalent resistance Rs and one single equivalent voltage Vs. Rs is the source resistance value looking back into the circuit and Vs is the open circuit voltage at the terminals.

Firstly, to analyse the circuit we have to remove the centre 40Ω load resistor connected across the terminals A-B, and remove any internal resistance associated with the voltage source(s). This is done by shorting out all the voltage sources connected to the circuit, that is v = 0, or open circuit any connected current sources making i = 0. The reason for this is that we want to have an ideal voltage source or an ideal current source for the circuit analysis.



The value of the equivalent resistance, Rs is found by calculating the total resistance looking back from the terminals A and B with all the voltage sources shorted. We then get the following circuit.



Find the Equivalent Resistance (RT)

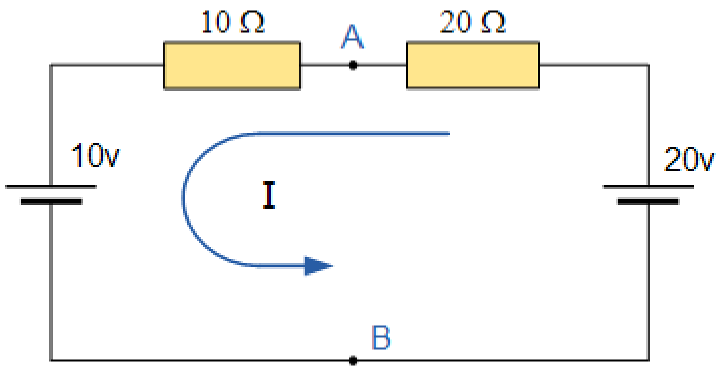
10Ω Resistor in Parallel with the 20Ω Resistor

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 10}{20 + 10} = 6.67\Omega$$

The voltage Vs is defined as the total voltage across the terminals A and B when there is an open circuit between them. That is without the load resistor R_L connected.

Find the Equivalent Voltage (V_T)

We now need to reconnect the two voltages back into the circuit, and as $V_T = V_{AB}$ the current flowing around the loop is calculated as:



$$I = \frac{V}{R} = \frac{20v - 10v}{20\Omega + 10\Omega} = 0.33\ amps$$

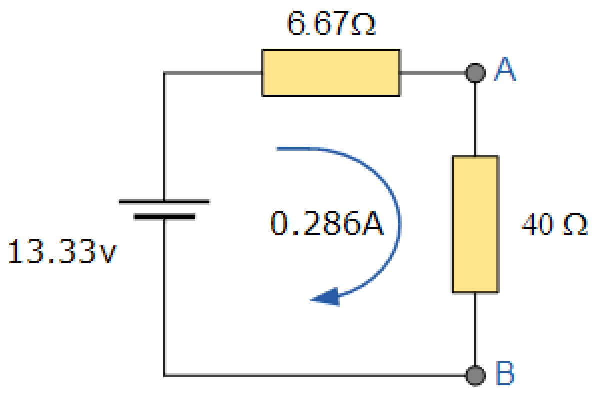
This current of 0.33 amperes (330mA) is common to both resistors so the voltage drop across the 20Ω resistor or the 10Ω resistor can be calculated as:

$$V_{AB} = 20 - (20\Omega \times 0.33amps) = 13.33 \text{ volts.}$$

or

$$V_{AB} = 10 + (10\Omega \times 0.33amps) = 13.33 \text{ volts, the same.}$$

Then the Thevenin's Equivalent circuit would consist or a series resistance of 6.67Ω and a voltage source of 13.33v. With the 40Ω resistor connected back into the circuit we get:



$$I = \frac{V}{R} = \frac{13.33v}{6.67\Omega + 40\Omega} = 0.286\ amps$$

Thevenin's Theorem Summary

We have seen here that Thevenin's theorem is another type of circuit analysis tool that can be used to reduce any complicated electrical network into a simple circuit consisting of a single voltage source, V_T in series with a single resistor, R_T.

When looking back from terminals A and B, this single circuit behaves in exactly the same way electrically as the complex circuit it replaces. That is the i-v relationships at terminals A-B are identical.

The basic procedure for solving a circuit using **Thevenin's Theorem** is as follows:

1. Remove the load resistor R_L or component concerned.

2. Find R_S by shorting all voltage sources or by open circuiting all the current sources.

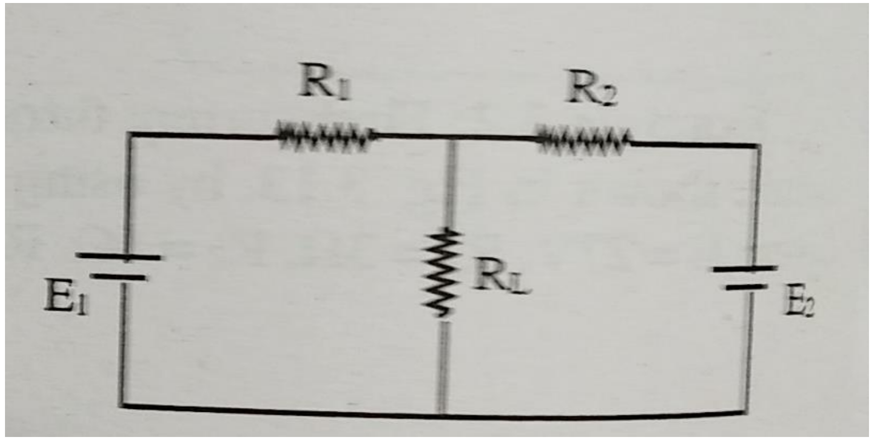
3. Find V_S by the usual circuit analysis methods.

4. Find the current flowing through the load resistor R_L.

Example : Using Thevenin's theorem find the current through load and potential difference across load in the given network, Given E_1 = 10V , E_2 = 20V , R_1 = 10 Ω , R_2 = 20 Ω , R_L = 40 Ω.

Solution :

Step 1: Calculation Of V_{TH} === Remove the load resistance and measure the open circuit voltage across the points A and B.



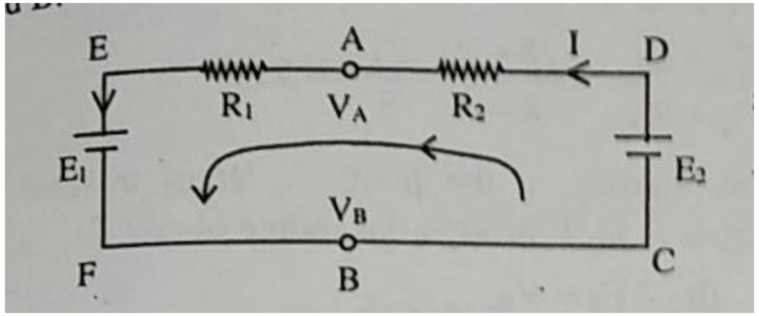
After removing R_L ; E_1 , E_2 , R_1 , R_2 are in series.

Applying KVL;

$E_2 - IR_2 - IR_1 - E_1 = 0$

$$I = \frac{E_2 \; - \; E_2}{R_2 + \; R_1} = \frac{20 \; - 10}{10 + 20} = 0.33A$$

Again KVL from A to B through E_1



$V_A - IR_1 - E_1 = V_B$

$V_A - V_B = E_1 + IR_1$

$V_{TH} = V_A - V_B = 10 + (0.33 \times 10) = 13.3$ V

Step 2 : Calculation of R_{TH}

Short circuit the voltage sources and remove the load R_L

$$R_{TH} = R_1 \;//\; R_2$$

$$= \frac{R_1 R_2}{R_1 + R_2} = \frac{200}{30} = 6.67\ \Omega$$

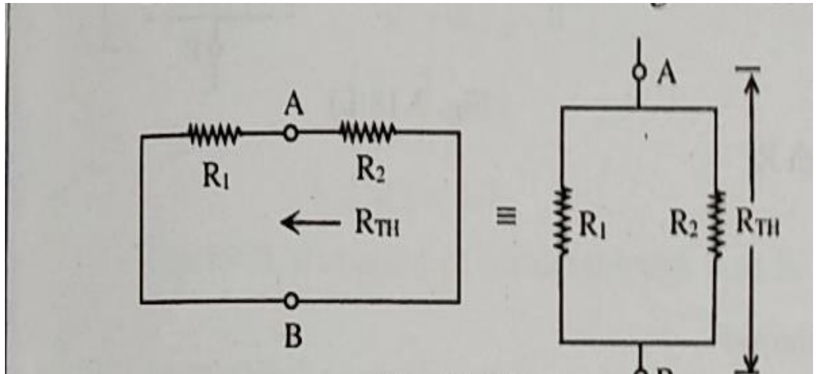
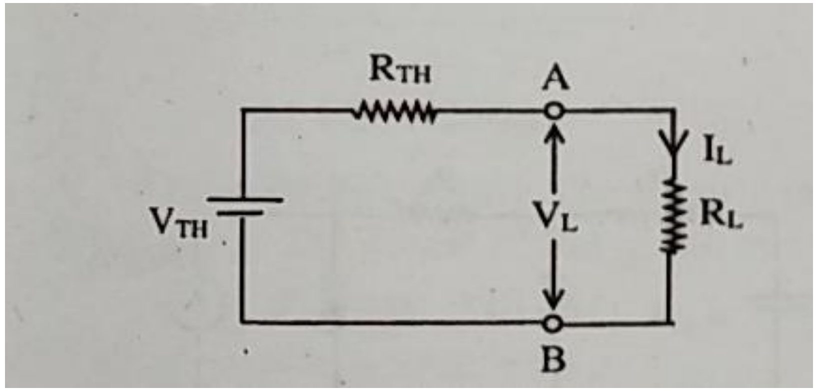


Fig. 3.16(b)

Step 3 : Calculation of load current and load voltage

$$I_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{13.3}{46.6} = 0.28\ A$$

$V_L = I_L R_L = 0.28 \text{ x } 40 = 11.2$ V



EXAMPLE : Apply Thevenin's theorem to find current through the load resistance of the circuit. E = 15 V, R_1 = 3 Ω , R_2 = 6 Ω , R_3 = 2 Ω , R_L = 6 Ω and constant current source 10A.

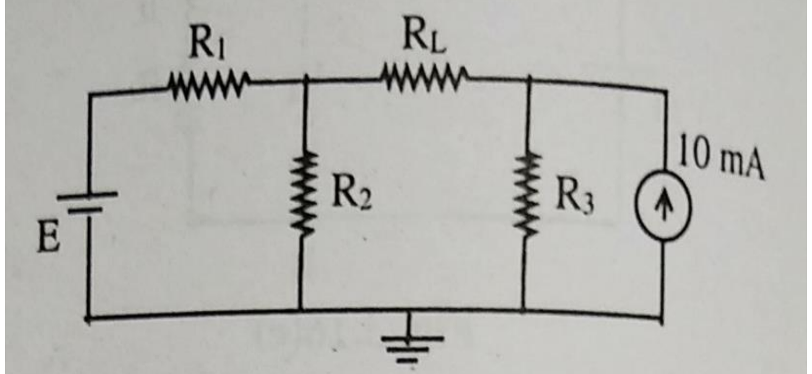


Fig. 3.19

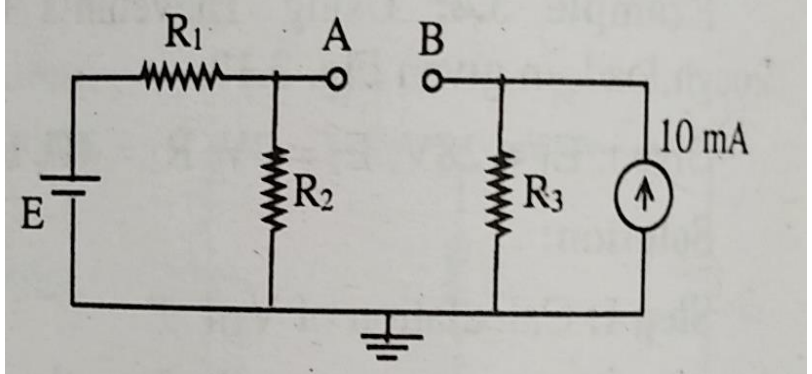


Fig. 3.20(a)

Solution :

Step 1 : Calculation of V_{TH} , Remove R_L and find the open circuit voltage between A and B.

The full current of 10A passes through the R_3 giving drop of 10 x 2 = 20V

Therefore V_B = 20 V.

15 V battery is connected in series with R_1 and R_2

V_A = voltage drop across the R_2 = $\left(\frac{E\ R_2}{R_1 + R_2}\right) = 10V$

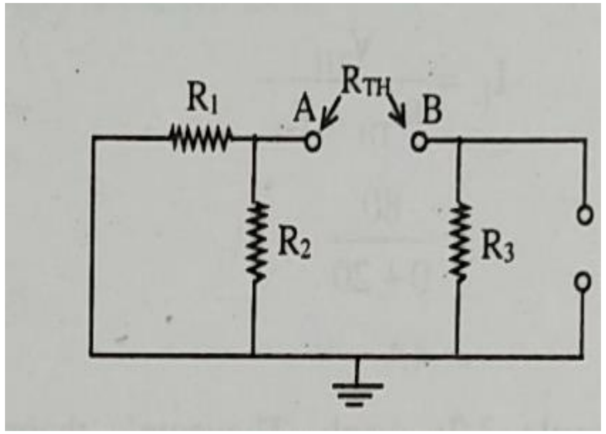
The potential drop between A and B = 20 – 10 = 10V.

Step 2: Calculation of R_{TH}

$$R_{TH} = (R1 // R_2) + R_3$$

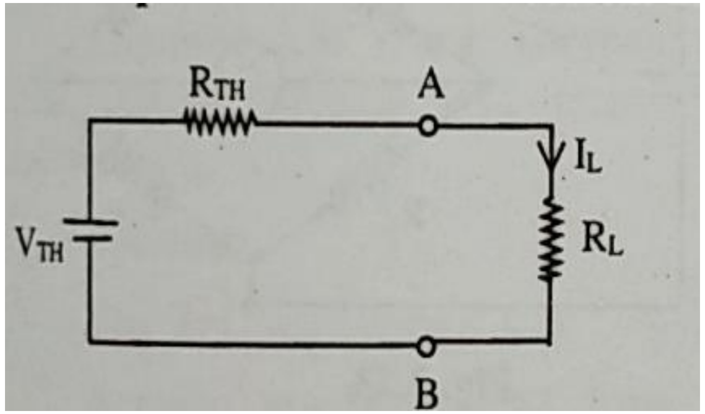
$$= \frac{R_1 R_2}{R_1 + R_2} + R_3$$

$$= \frac{3 \, x \, 6}{3+6} + 2 = 4 \, \Omega$$



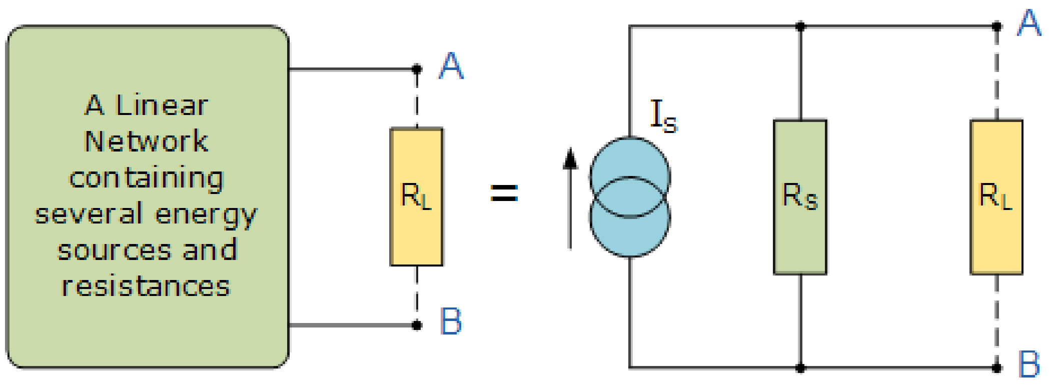
Step 3 : Calculation of I_L

$$I_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{10}{4+6} = 1 \, A$$

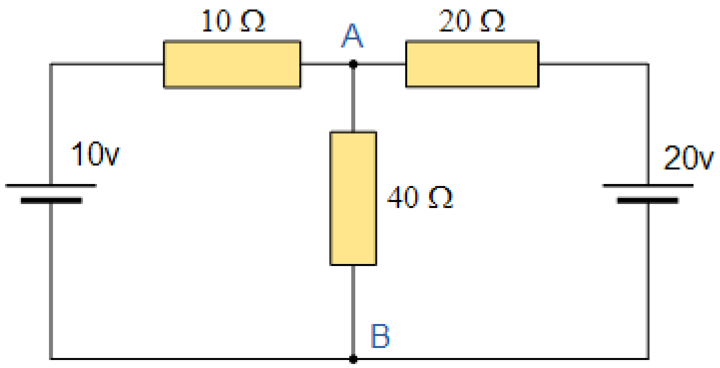


Norton's Theorem

- Norton on the other hand reduces his circuit down to a single resistance in parallel with a constant current source.
- **Nortons Theorem** states that "*Any linear circuit containing several energy sources and resistances can be replaced by a single Constant Current generator in parallel with a Single Resistor*".
- As far as the load resistance, R_L is concerned this single resistance, R_S is the value of the resistance looking back into the network with all the current sources open circuited and I_S is the short circuit current at the output terminals as shown below.

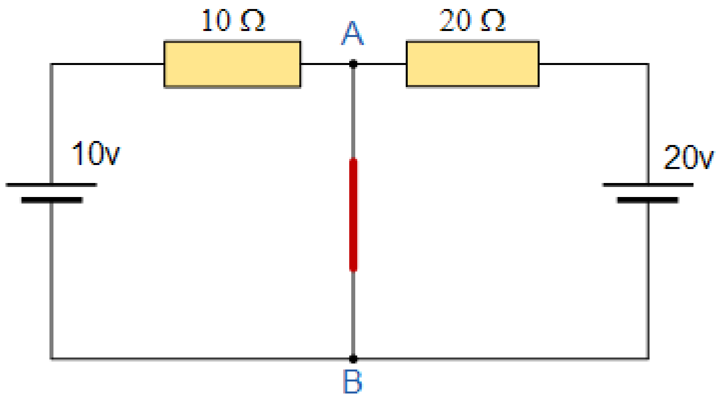


Example



To find the Norton's equivalent of the above circuit we firstly have to remove the center 40Ω load resistor and short out the terminals A and B to give us the following circuit.

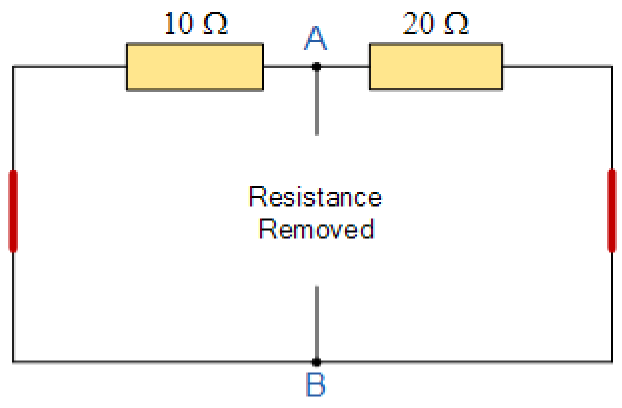
When the terminals A and B are shorted together the two resistors are connected in parallel across their two respective voltage sources and the currents flowing through each resistor as well as the total short circuit current can now be calculated as:



$$I_1 = \frac{10v}{10\Omega} = 1amp, \quad I_2 = \frac{20v}{20\Omega} = 1amp$$

$$\text{therefore, } I_{\text{short-circuit}} = I_1 + I_2 = 2amps$$

If we short-out the two voltage sources and open circuit terminals A and B, the two resistors are now effectively connected together in parallel. The value of the internal resistor Rs is found by calculating the total resistance at the terminals A and B giving us the following circuit.

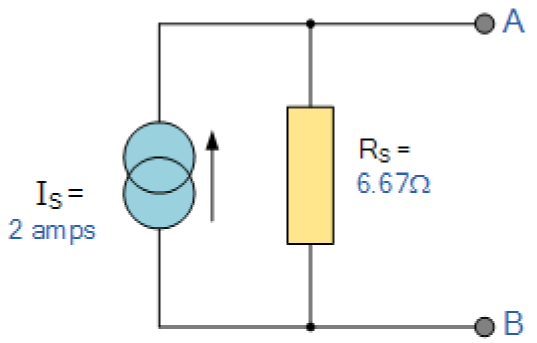


Find the Equivalent Resistance (RN)

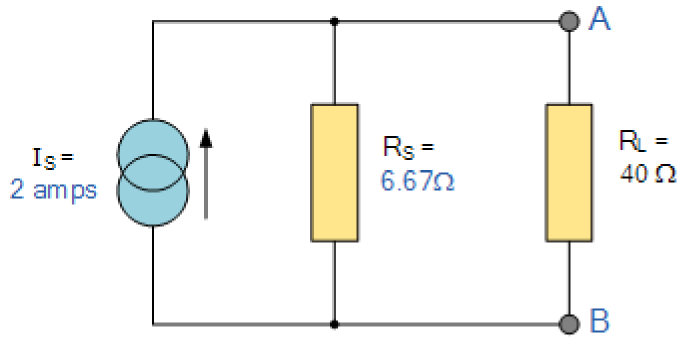
$$10\Omega \text{ Resistor in Parallel with the } 20\Omega \text{ Resistor}$$

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 10}{20 + 10} = 6.67\Omega$$

Having found both the short circuit current, Is and equivalent internal resistance, Rs this then gives us the following **Norton's equivalent circuit**.



Ok, so far so good, but we now have to solve with the original 40Ω load resistor connected across terminals A and B as shown below.



Again, the two resistors are connected in parallel across the terminals A and B which gives us a total resistance of:

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{6.67 \times 40}{6.67 + 40} = 5.72\Omega$$

The voltage across the terminals A and B with the load resistor connected is given as: $V_{A\text{-}B} = I \times R = 2 \times 5.72 = 11.44v$

Then the current flowing in the 40Ω load resistor can be found as:

$$I = \frac{V}{R} = \frac{11.44}{40} = 0.286 \text{ amps}$$

Norton's Theorem Summary

The basic procedure for solving a circuit using **Norton's Theorem** is as follows:

1. Remove the load resistor R_L or component concerned.

2. Find R_S by shorting all voltage sources or by open circuiting all the current sources.

3. Find I_S by placing a shorting link on the output terminals A and B.

4. Find the current flowing through the load resistor R_L.

Example : Norton's Theorem

Find the current through the load resistance of the given network. Usw Norton's theorem. Given E = 9 V , R_1 = 3Ω , R_2 = 6Ω , R_3 = 6Ω , R_4 = 3Ω, R_L = 3Ω .

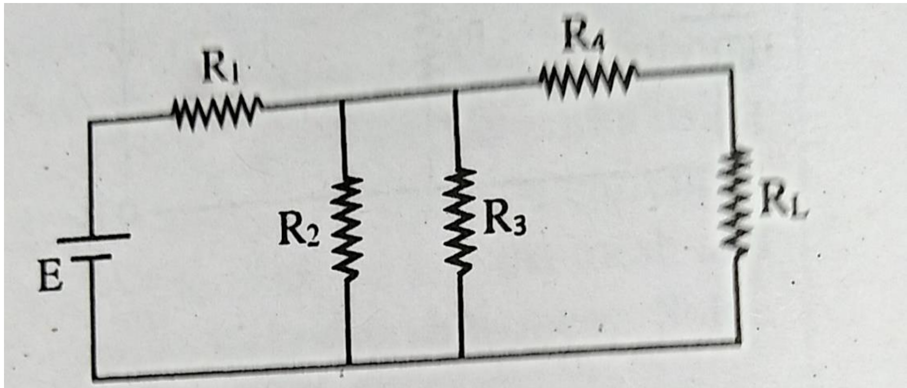


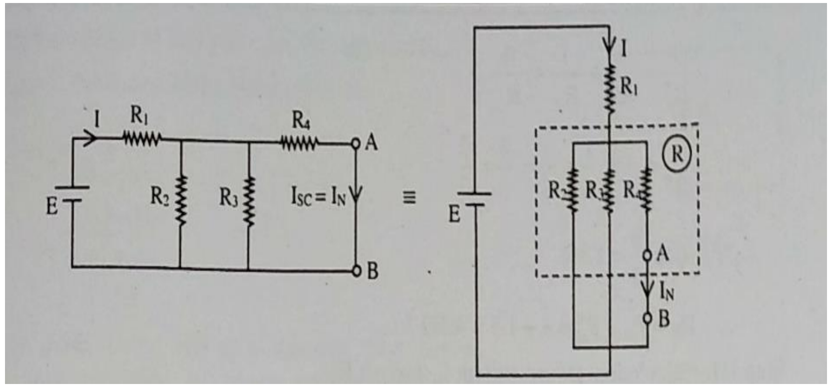
Fig. 3.34

Solution :

Step 1 Calculation of I_N

Replace R_L with short circuit.

Current passing through R_4 is a short circuit current $I_{SC} = I_N$



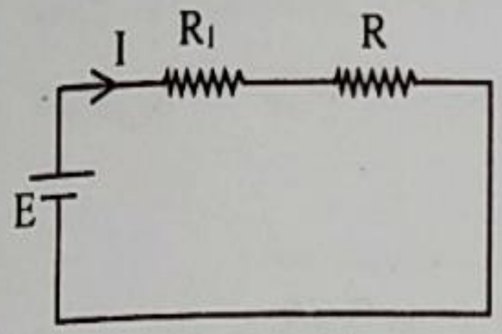


Fig. 3.35(b)

$$R = R_2 \parallel R_3 \parallel R_4$$

$$\therefore \quad \frac{1}{R} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\therefore \quad \frac{1}{R} = \frac{1}{6} + \frac{1}{6} + \frac{1}{3} = \frac{4}{6} = \frac{2}{3}$$

$$\therefore \quad R = \frac{3}{2} = 1.5\Omega$$

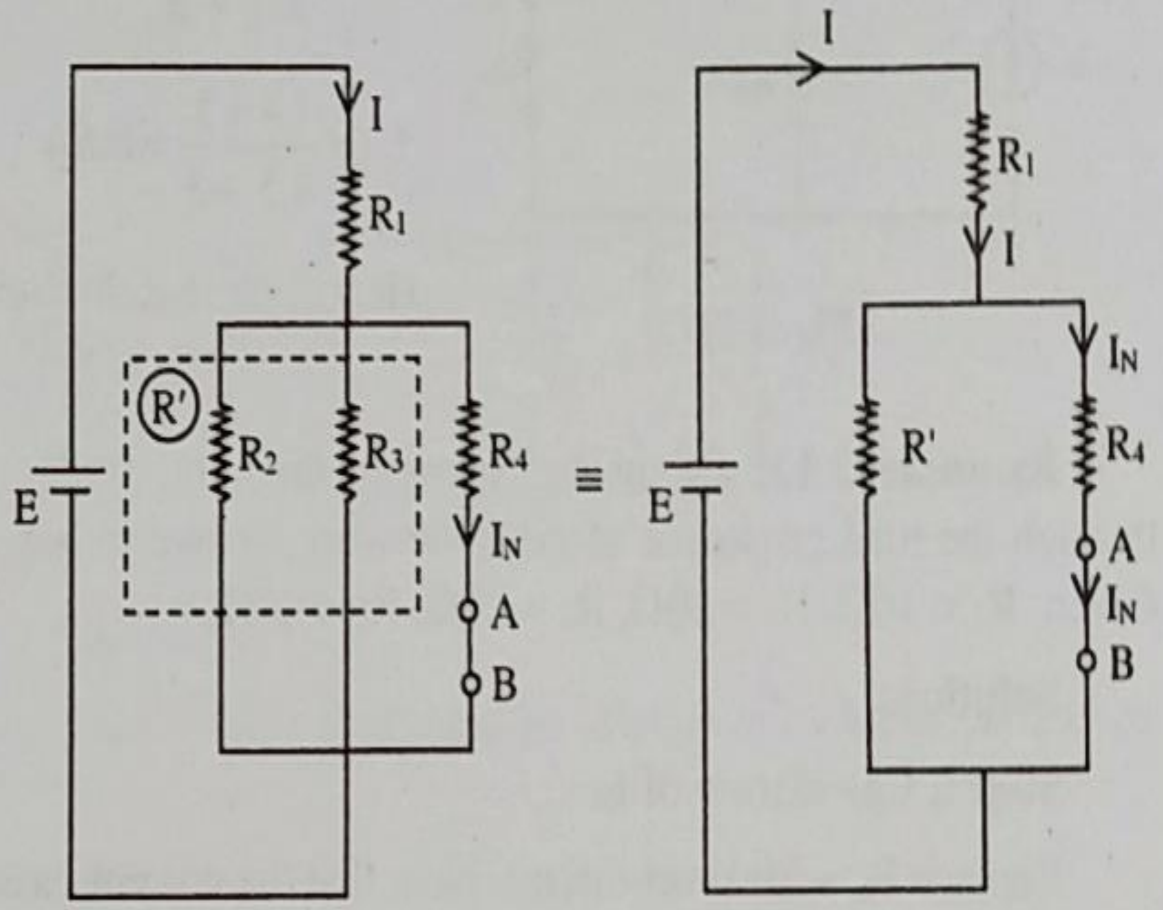


Fig. 3.35(c)

$$\therefore I = \frac{E}{R_1 + R} = \frac{9}{3 + 1.5} = 2A$$

Equivalent resistance ;

$$R^/ = R_2 \;\; // \;\; R_3$$

$$\frac{1}{R^/} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}\Omega$$

$$R^/ = 3\ \Omega$$

$$I_{SC} = I_N\ I\left(\frac{R^/}{R^/ + R_4}\right)$$

Substituting the values

$I_{SC} = I_N = 3$ A

Calculation of R_N :

Since battery has no internal resistance it is replaced by a short circuit, The resistance R_N of the circuit is resistance between terminals A and B;

$R^{//} = R_1 \; // \; R_2 \; // \; R_3$

$1/R^{//} = 1/3 + 1/6 + 1/6 = 2/3$

$R^{//} = 3/2 = 1.5\ \Omega$

$R_N = R_4 + R^{//} = 3 + 1.5 = 4.5\ \Omega$

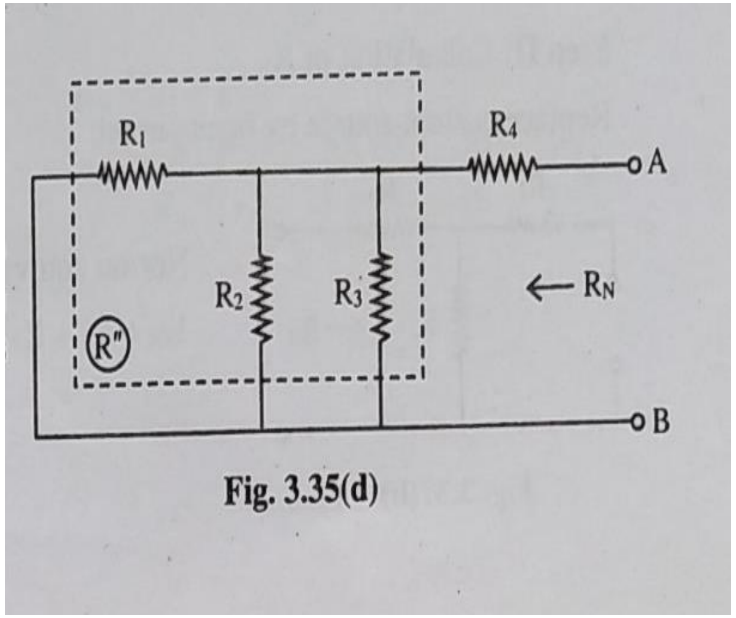
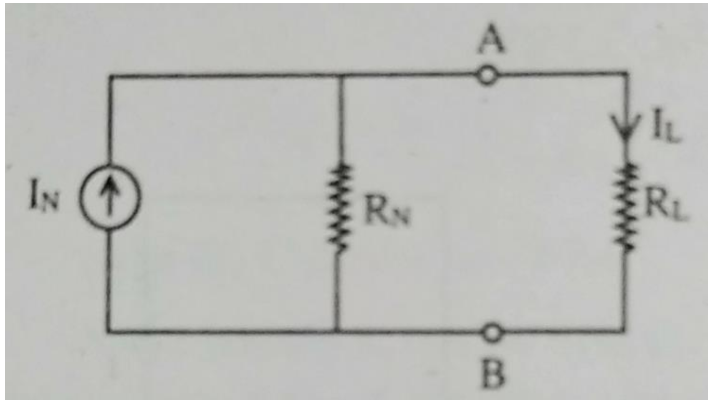


Fig. 3.35(d)

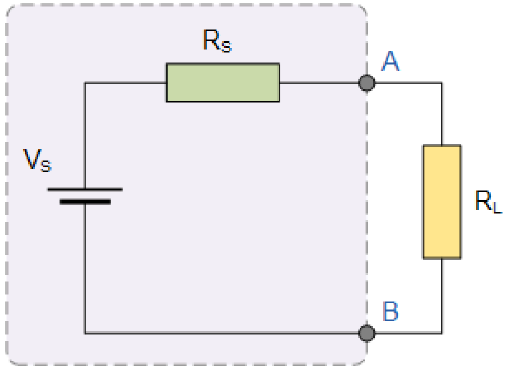
Step 3: Calculation of I_L

$$I_L = \frac{I_N\, R_L}{R_N + R_L} = \frac{1 \times 4.5}{4.5 + 3} = 0.6\ A$$



Maximum Power Transfer Theorem

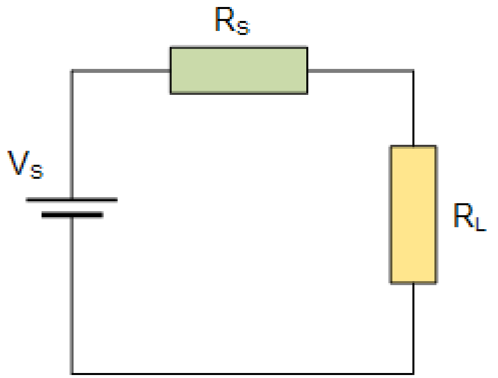
The **Maximum Power Transfer Theorem** is another useful circuit analysis method to ensure that the maximum amount of power will be dissipated in the load resistance when the value of the load resistance is exactly equal to the resistance of the power source. The relationship between the load impedance and the internal impedance of the energy source will give the power in the load. Consider the circuit below.



- In our Thevenin equivalent circuit above, the maximum power transfer theorem states that "*the maximum amount of power will be dissipated in the load resistance if it is equal in value to the Thevenin or Norton source resistance of the network supplying the power*".
- In other words, the load resistance resulting in greatest power dissipation must be equal in value to the equivalent Thevenin source resistance, then $R_L = R_S$ but if the load resistance is lower or higher in value than the Thevenin source resistance of the network, its dissipated power will be less than maximum.

.

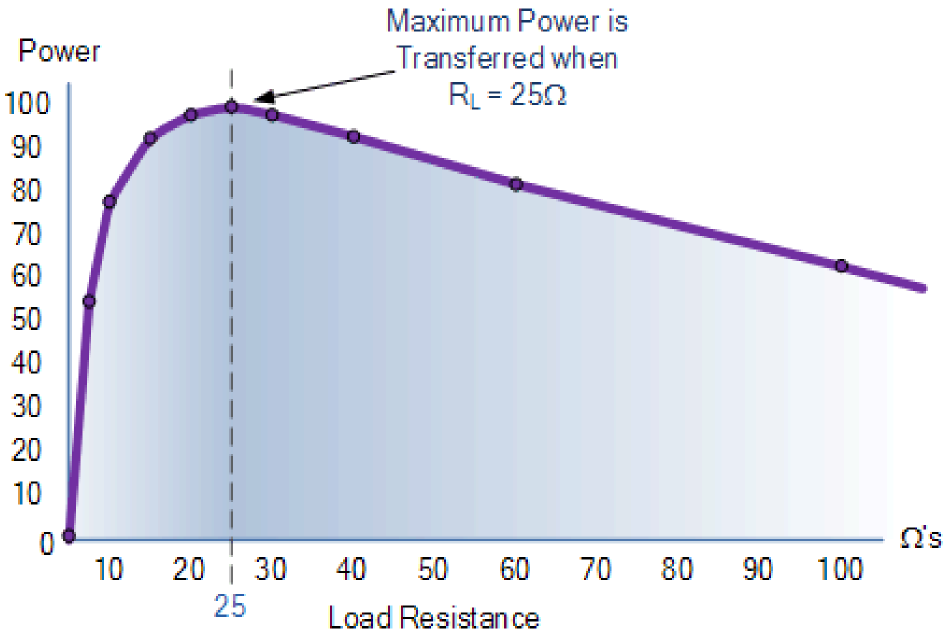
For example, find the value of the load resistance, R_L that will give the maximum power transfer in the following circuit



Then by using the following Ohm's Law equations:

$$I = \frac{V_S}{R_S + R_L} \quad \text{and} \quad P = I^2 R_L$$

R_L (Ω)	I (amps)	P (watts)
0	4.0	0
5	3.3	55
10	2.8	78
15	2.5	93
20	2.2	97
R_L (Ω)	I (amps)	P (watts)
25	2.0	**100**
30	1.8	97
40	1.5	94
60	1.2	83
100	0.8	64

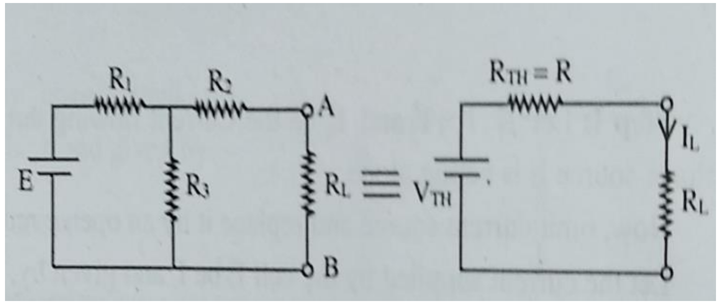


From the above table and graph we can see that the **Maximum Power Transfer** occurs in the load when the load resistance, R_L is equal in value to the source resistance, R_S that is: $R_S = R_L = 25\Omega$. This is called a "matched condition" and as a general rule, maximum power is transferred from an active device such as a power supply or battery to an external device when the impedance of the external device exactly matches the impedance of the source.

Proof : Maximum power transfer theorem

Consider the given network.

Convert the n/w into Thevenin equivalent n/w containing V_{Th} and R_{TH}



The load current is; $I_L = \frac{V_{TH}}{R_{TH} + R_L}$

The load voltage is ; $V_L = \frac{V_{TH}\, R_L}{R_{TH} + R_L}$

The power delivered to the load ; $P = I_L^2 \ R_L = \frac{V_{TH}^2\, R_L}{(R_{TH} + R_L)^2}$

To determine the value of R_L for maximum power to be transferred to the load,

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left[\frac{V_{TH}^2 R_L}{((R_{Th} + R_L)^2)}\right] = V_{TH}^2 \left[\frac{((R + R_L)^2) - 2R_L\ (R + R_L\ )}{(R + R_L)^2}\right] = 0$$

$$\therefore V_{TH}^2 \left[\frac{((R + R_L)^2) - 2R_L\ (R + R_L\ )}{(R + R_L)^2}\right] = 0$$

$$((R + R_L)^2) - 2R_L\ (R + R_L\ ) = 0$$

$$\therefore\ R^2 - R_L^2 = 0$$

$R = R_L$ or

$R_{TH} = R_L$

Thus, the maximum power will be delivered to the load when load resistance is equal to the internal resistance of the n/w delivering the power.

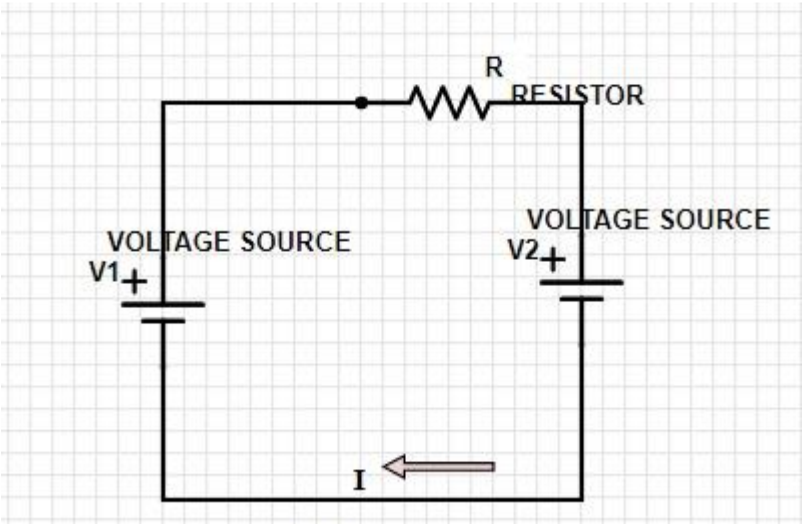
Super position theorem

Superposition theorem is based on the concept of linearity between the response and excitation of an electrical circuit. It states that the response in a particular branch of a linear circuit when multiple independent sources are acting at the same time is equivalent to the sum of the responses due to each independent source acting at a time.

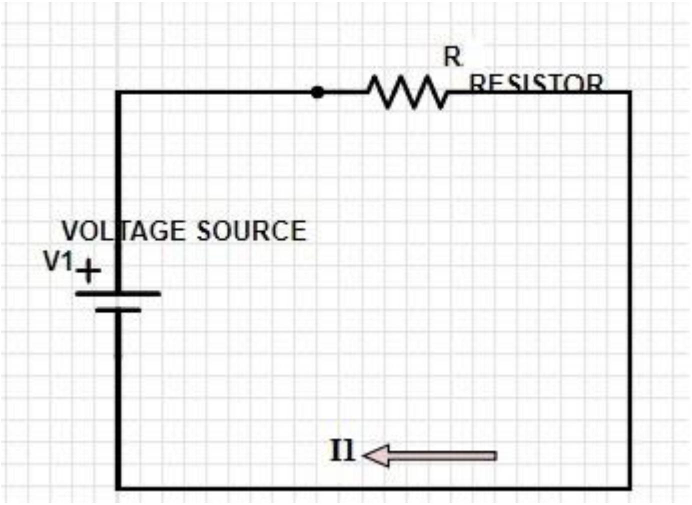
In this method, we will consider only **one independent source** at a time. So, we have to eliminate the remaining independent sources from the circuit. We can eliminate the voltage sources by shorting their two terminals and similarly, the current sources by opening their two terminals.

Therefore, we need to find the response in a particular branch **‘n’ times** if there are ‘n’ independent sources. The response in a particular branch could be either current flowing through that branch or voltage across that branch.

The basic circuit diagram of superposition theorem is shown below, and it is the best example of this theorem. By using this circuit, calculate the flow of current through the resistor R for the following circuit.



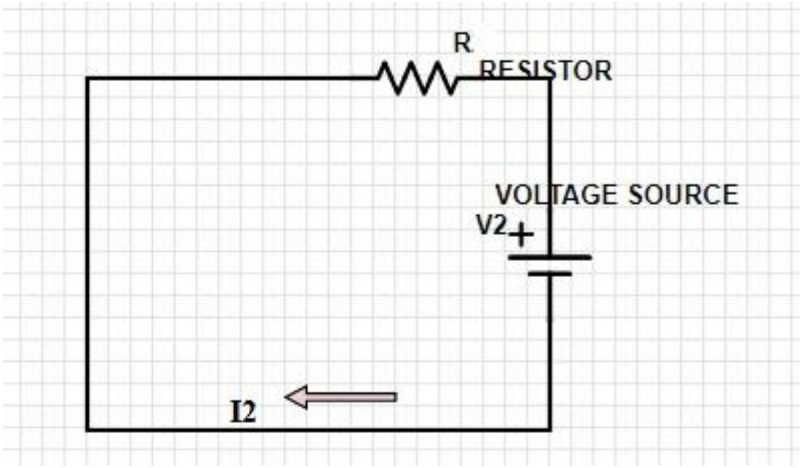
Disable the secondary voltage source i.e, V2, and calculating the flow of current I1 in the following circuit.



We know that ohms law V= IR

I1= V1/R

Disable the primary voltage source i.e, V1, and calculating the flow of current I2 in the following circuit.



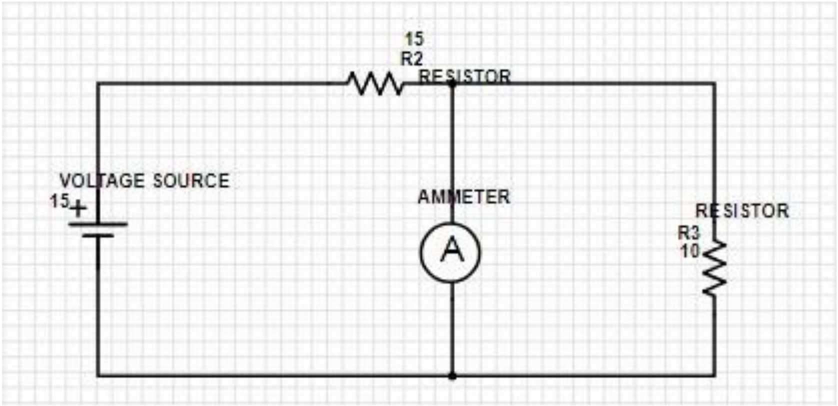
I2= -V2/R

According to superposition theorem, the network current I = I1 + I2

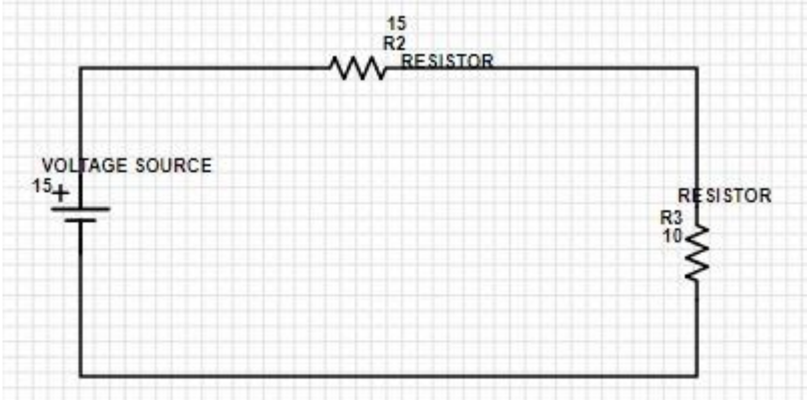
I = V1/R-V2/R

Superposition Theorem Problems

The following circuit shows the basic DC circuit for solving the superposition theorem problem such that we can get the voltage across the load terminals. In the following circuit, there are two independent supplies namely current and voltage



Initially, in the above circuit, we keep only voltage supply is acting, and the remaining supply like the current is changed with inside resistance. So the above circuit will become an open circuit as shown in the below figure.



Consider the voltage across the load terminals VL1 with voltage supply performing alone, then

VL1 =Vs (R3/(R3 + R1))

Here, Vs= 15, R3= 10 and R2-= 15

Please substitute the above values in the above equation

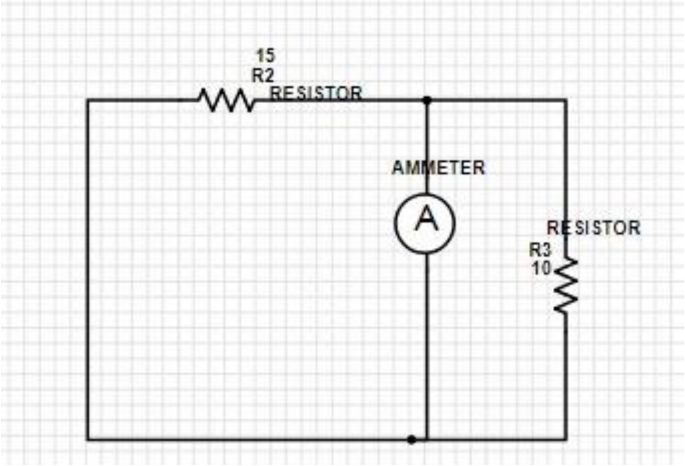
VL1 = Vs × R3 / (R3 + R2)

= 15 (10 / (10 + 15))

= 15(10/25)

= 6 Volts

Hold the current supply only and change the voltage supply with its inside resistance. So the circuit will become a short circuit as shown in the following figure.



Consider the voltage across the load terminals is 'VL2' while only current supply performing. Then

VL2= I x R

IL = 1 x R1/(R1+R2)

R1 = 15 RL= 25

= 1 × 15 / (15 +25) = 0.375 Amps

VL2 = 0.375 × 10 = 3.75 Volts

As a result, we know that the superposition theorem states that the voltage across the load is the amount of VL1 & VL2

VL = VL1 + VL2 = 6 + 3.75 = 9.75 Volts

Applications & Limitations of Superposition Theorem

- The superposition theorem cannot be useful for power calculations but this theorem works on the principle of linearity. As the power equation is not linear. As a result, the power used by the factor in a circuit with this theorem is not achievable.
- If the load selection is changeable, then it is necessary to achieve each supply donation and their calculation for each transform in load resistance. So this is a very difficult method to analyze compound circuits.
- The **application of superposition theorem** is, we can employ only for linear circuits as well as the circuit which has more supplies.
- From the above superposition theorem examples, this theorem cannot be used for non-linear circuits, but applicable for linear circuits. The circuit can be examined with the single power source at a time, the
- Equivalent section currents and voltages algebraically included discovering what they will perform with every power supplies in effect. To cancel out all except one power supply for study, substitute any power source with a cable; restore any current supply with the break.

THANK YOU